Appendix C1 – Derivation of the Elastic Parameters for shadecloth

Elastic Theory

The general relationship between stress and strain (Hooke's law) for knitted shadecloth, assumed to have different elastic response in each direction and negligible shear stress, can be expressed in two dimensional matrix form as follows:-

$$\begin{bmatrix} \sigma_{ww} \\ \sigma_{ff} \end{bmatrix} = \begin{bmatrix} E_{ww} & E_{wwff} \\ E_{ffww} & E_{ff} \end{bmatrix} . \begin{bmatrix} \epsilon_{ww} \\ \epsilon_{ff} \end{bmatrix}$$

where σ_{ww} and σ_{ff} are the stresses; ϵ_{ww} and ϵ_{ff} the strains; in the warp and fill (i.e. weft) directions; and with the "hindered" stiffness matrix being

$$E = \begin{bmatrix} E_{ww} & E_{wwff} \\ E_{ffww} & E_{ff} \end{bmatrix}$$

The term "hindered" describes the situation where the strain response induced in the lateral direction is restrained, such as in a biaxial test.

The matrix relationship can be re-expressed as linear equations:-

$$\sigma_{ww} = E_{ww}.\epsilon_{ww} + E_{wwff}.\epsilon_{ff} \qquad \text{-------- equation (1)}$$

$$\sigma_{ff} = E_{ffww}.\epsilon_{ww} + E_{ff}.\epsilon_{ff} \qquad \text{-------- equation (2)}$$

The values E_{ww} and E_{ff} are not the same as the Elastic Moduli, namely E_w and E_f which are ''unhindered", where the strain response in the lateral direction is not restrained, such as in a uniaxial test. The relationship between the "hindered" and "unhindered" values can be derived from the Compliance matrix "C", which is the inverse of the Stiffness Matrix.

$$\begin{bmatrix} \epsilon_{ww} \\ \epsilon_{ff} \end{bmatrix} = \begin{bmatrix} C_{ww} & C_{wwff} \\ C_{ffww} & C_{ff} \end{bmatrix} . \begin{bmatrix} \sigma_{ww} \\ \delta_{ff} \end{bmatrix}$$

For orthotropic materials (i.e. different properties in each direction)

$$C = \begin{bmatrix} 1/E_w & -\nu_{fw}/E_f \\ -\nu_{wf}/E_w & 1/E_f \end{bmatrix}$$

Since $E = C^{-1}$, then:-

$$\begin{bmatrix} E_{ww} & E_{wwff} \\ E_{ffww} & E_{ff} \end{bmatrix} = \begin{bmatrix} C_{ww} & C_{wwff} \\ C_{ffww} & C_{ff} \end{bmatrix}^{-1}$$

and by inverting *C* above:-

$$C^{-1} = \begin{bmatrix} \frac{E_w}{1-\nu_{wf}\nu_{fw}} & \frac{\nu_{fw}E_w}{1-\nu_{wf}\nu_{fw}} \\ \frac{\nu_{wf}E_f}{1-\nu_{wf}\nu_{fw}} & \frac{E_f}{1-\nu_{wf}\nu_{fw}} \end{bmatrix}$$

Hence $E_{ww} = \frac{E_w}{1-\nu_{wf}\nu_{fw}}$ and $E_{ff} = \frac{E_f}{1-\nu_{wf}\nu_{fw}}$ ------ equations (3) and (4)

It is a property of Elastic materials that the matrix is symmetric, i.e. $E_{ffww} = E_{wwff}$ and therefore:-

$$\frac{E_w}{\nu_{wf}} = \frac{E_f}{\nu_{fw}} \qquad \text{----- equation (5)}$$

Substituting equation (5) into (3) and (4) respectively:-

$$\nu_{wf} = \sqrt{\left(1 - {}^{E_w}\!/_{E_{ww}}\right).{}^{E_w}\!/_{E_f}} \quad \text{------------ Equation (6)}$$

And also:-

$$\nu_{fw} = \sqrt{\left(1 - {}^{E_f}\!/_{E_{ff}}\right).{}^{E_f}\!/_{E_w}} \quad \text{------------Equation (7)}$$

With simplifying assumptions, the four variables E_{ww}, E_w , E_{ff} and E_f for the Polyfab ranges can be derived from the Uniaxial and Biaxial test results; and the Poisson's ratios ν_{wf} , and ν_{fw} calculated by substitution in the equations (6) and (7) above.

The standard uniaxial tests provide only the load and extension at failure; but the graphics associated with the test confirmed that an approximate linear relationship exists between the two. Thus, the slopes (i.e. E_w and E_f) can be calculated from the starting co-ordinate and the load and extension at break reported in the standard test.

The test data reported in Table One is derived from the Australian Standard AS 2001.2.3.1 – 2001. The test is carried out on a strip of 50 mm width, with a pre-tension load of five Newtons. Thus, the starting and finishing co-ordinates of the tests can be determined, and the Modulus is simply the slope of the line between the two points.

The "hindered" values (i.e. E_{ww} and E_{ff}) need to be derived iteratively, assuming these slopes must lie between those of the uniaxial tests, and those of the Biaxial test results fitted using the least squares method. This latter slope represents the best actual fit to the test results, and the stresses calculated from this linear equation should match as closely as possible the outcome of applying equations (1) and (2).

However, the procedure needs to be used with caution, as the fitting of the curves is approximate at best, and there is a range of possible estimates for E_{ww} and E_{ff}, particularly if there is non-compliance with the constraint $\frac{E_w}{\nu_{wf}} = \frac{E_f}{\nu_{fw}}$.

For these and similar reasons, the method should be considered a guide only and engineers should apply their own best judgement in deriving the parameters.

Typical results for Polyfab Comshade using the procedure is shown in the following table, with the graphics below:-

Range	E_w (kN/m)	E_f (kN/m)	E_{ww}(kN/m)	E_{ff}(kN/m)	ν_{wf}	ν_{fw}
Comshade	34.0	70.8	37.4	123.8	0.33	0.69